SET THEORY

1. Write in words how to read each of the following.
a) $\{x \in R+ \mid 0 < x < 1\}$
b) $\{x \in R \mid x \leq 0 \text{ or } x \geq 1\}$
c) $\{n \in Z \mid n \text{ is a factor of } 6\}$
d) $\{n \in Z+ \mid n \text{ is a factor of } 6\}$

2. Answer the following
a) Is $4 = \{4\}$?
b) How many elements are in the set $\{3, 4, 3, 5\}$?
c) How many elements are in the set $\{1, \{1\}, \{1, \{1\}\}\}$?

3. Which of the following sets are equal?
$A = \{0, 1, 2\}$
$B = \{x \in R \mid -1 \leq x < 3\}$
$C = \{x \in R \mid -1 < x < 3\}$
$D = \{x \in Z \mid -1 < x < 3\}$
$E = \{x \in Z+ \mid -1 < x < 3\}$

4. Answer the following
a) Is $3 \in \{1, 2, 3\}$?
b) Is $1 \subseteq \{1\}$?
c) Is $\{2\} \in \{1, 2\}$?
d) Is $\{3\} \in \{1, \{2\}, \{3\}\}$?
e) Is $\{2\} \subseteq \{1, \{2\}, \{3\}\}$?
f) Is $\{1\} \subseteq \{1, 2\}$?
g) Is $1 \in \{\{1\}, 2\}$?
h) Is $\{1\} \subseteq \{1, \{2\}\}$?
i) Is $\{1\} \subseteq \{1\}$?

5. Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$.
Answer each of the following questions. Give reasons for your answers.
a) Is $B \subseteq A$?
b) Is $C \subseteq A$?
c) Is $C \subseteq C$?
d) Is C a proper subset of A?

6. Let $S = \{2, 4, 6\}$ and $T = \{1, 3, 5\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:
a) $S \times T$
b) $T \times S$
c) $S \times S$
d) $T \times T$

7. In a class of 80 students, 50 students knows English, 55 knows French, 46 knows German and 37 students knows English & French, 28 students knows French & German, 7 students knows none of the languages. Find out how many students knows all three languages.

8. State Inclusion Exclusion Principle and hence find how many integers in $\mathbb{N}_{500}$ is not divisible by **4** or **7**?

9. A survey of 550 T.V watchers produced the following information:
285 watch football game, 195 watch hockey game, 115 watch baseball game, 45 watch football and baseball game, 70 watch hockey and baseball game, 35 watch football and hockey 100 do not watch any of the game.
i) How many people in the survey watch all 3 games?
ii) How many people watch exactly one of the three game?

LOGIC

1. Write the statements in symbolic form using the symbols ∼,∨, and ∧ and the indicated letters to represent component statements.
Let p: "John is healthy," q: "John is wealthy," and s: "John is wise."

a) John is healthy and wealthy but not wise.
b) John is not wealthy but he is healthy and wise.
c) John is neither healthy, wealthy, nor wise.
d) John is neither wealthy nor wise, but he is healthy.
e) John is wealthy, but he is not both healthy and wise.

2. Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentences in symbolic notation.

a) DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.
b) DATAENDFLAG is off but ERROR is not equal to 0.
c) DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.
d) DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.
e) Either DATAENDFLAG is on or it is the case that both
f) ERROR equals 0 and SUM is less than 1,000.

3. Check whether following statement are tautology or contradiction

a) $(p \rightarrow q) \leftrightarrow \sim (p \vee q)$
b) $(p \wedge (p \rightarrow q)) \rightarrow q$

4. Show that the following statement is logically equivalent.

a) $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$
b) $p \rightarrow q \vee r \equiv [(p \vee r) \rightarrow (q \vee r)]$

5. Write negations, contrapositives and converse for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

a) If P is a square, then P is a rectangle.
b) If today is New Year's Eve, then tomorrow is January.
c) If the decimal expansion of r is terminating, then r is rational.
d) If n is prime, then n is odd or n is 2.
e) If x is nonnegative, then x is positive or x is 0.
f) If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
g) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

6. Let P(x) be the predicate "x > 1/x."
Write P (2), P (1/2), P (−1), P (−1/2), and P (−8), and indicate which of these statements are true and which are false.

7. Let Q(x, y) be the predicate "If x < y then x2 < y2" with domain for both x and y being the set R of real numbers.

a) Explain why Q(x, y) is false if x = −2 and y = 1.
b) Give values different from those in part (a) for which Q(x, y) is false.
c) Explain why Q(x, y) is true if x = 3 and y = 8.
d) Give values different from those in part (c) for which Q(x, y) is true.