

1. Use the set-roster notation to indicate the elements in each of the following sets.
 - i. $S = \{n \in \mathbb{Z} \mid n = (-1)^k, \text{ for some integer } k\}$.
 - ii. $T = \{m \in \mathbb{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\}$.
 - iii. $U = \{r \in \mathbb{Z} \mid 2 \leq r \leq -2\}$
 - iv. $V = \{s \in \mathbb{Z} \mid s > 2 \text{ or } s < 3\}$
 - v. $W = \{t \in \mathbb{Z} \mid 1 < t < -3\}$
 - vi. $X = \{u \in \mathbb{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

2. Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$.

Answer each of the following questions. Give reasons for your answers.

- a. Is $B \subseteq A$?
- b. Is $C \subseteq A$?
- c. Is $C \subseteq C$?
- d. Is C a proper subset of A ?

3. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$.

A. Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?

B. Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?

4. Use an element argument to prove each statement

A. For all sets A, B , and C , $(A - B) \cup (C - B) = (A \cup C) - B$.

B. For all sets A, B , and C , $(A - B) \cap (C - B) = (A \cap C) - B$.

5. Write a negation for each of the following statements. Indicate which is true, the statement or its negation. Justify your answers.

a. \forall sets S, \exists a set T such that $S \cap T = \emptyset$.

b. \exists a set S such that \forall sets $T, S \cup T = \emptyset$.

6. Verify whether the given statement is True or False

For all sets A, B , and $C, A \cap (B - C) = (A \cap B) - (A \cap C)$.

7. Let $h =$ “John is healthy,” $w =$ “John is wealthy,” and $s =$ “John is wise.”

Write the following into symbolic form

a. John is healthy and wealthy but not wise.

d. John is not wealthy but he is healthy and wise.

b. John is neither healthy, wealthy, nor wise.

e. John is neither wealthy nor wise, but he is healthy.

c. John is wealthy, but he is not both healthy and wise.

8. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

i. If P is a square, then P is a rectangle.

ii. If today is New Year’s Eve, then tomorrow is January.

iii. If n is prime, then n is odd or n is 2.

iv. If x is nonnegative, then x is positive or x is 0.

v. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

9. Determine whether the statements in (a) and (b) are logically equivalent.

A. Assume x is a particular real number.

a. $x < 2$ or it is not the case that $1 < x < 3$.

b. $x \leq 1$ or either $x < 2$ or $x \geq 3$.

B. $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

10. Define the following:

a. Proposition

d. Valid arguments

b. Null set

e. Disjoint set

c. Tautology

f. De-morgan’s Law