MEASURES OF DISPERSION

- The **Measures of central tendency** gives us a birds eye view of the entire data they are called **averages of the first order,** it serve to locate the center of the distribution but they do not reveal how the items are spread out on either side of the central value. The measure of the scattering of items in a distribution about the average is called dispersion.

- Dispersion measures the extent to which the items vary from some central value. It may be noted that the measures of dispersion or variation measure only the degree but not the direction of the variation. The measures of dispersion are also called **averages of the second order** because they are based on the deviations of the different values from the mean or other measures of central tendency which are called averages of the first order.
- In the words of Bowley "**Dispersion is the measure of the variation of the items**"
- According to Conar "**Dispersion is a measure of the extent to which the individual items vary**"
- According to Reiglemen, **"Dispersion is the extent to which the magnitudes or quantities of the items differ, the degree of diversity."**

  The word dispersion may also be used to indicate the spread of the data.

METHODS OF MEASURING DISPERSION

- Range
- Quartile Deviation
- Mean Deviation
- Standard Deviation

Absolute measures of dispersion	**Relative measures of dispersion**
Range (R)	Coefficient of Range (CR)
Quartile Deviation (Q.D.)	Coefficient of Quartile Deviation (C.Q.D.)
Mean Deviation (M.D.)	Coefficient of Mean Deviation (C.M.D.)
Standard Deviation (S.D.)	Coefficient of Variation (C.V.)

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- **Range:**

It is defined as the difference between the smallest and the largest observations in a given set of data.

R = L - S Where, L: largest Value, S: Smallest value
Coefficient of Range: CR= $\frac{L-S}{L+S}$

- **Quartile Deviation:**

It is based on the quartiles so while calculating this may require upper quartile (Q_3) and lower quartile (Q_1) and then is divided by 2. Hence it is half of the deference between two quartiles it is also a semi inter quartile range.

Q.D. = $\frac{Q_3-Q_1}{2}$
$Q_1 = l + \frac{\left(\frac{N}{4}-c.f.\right)*h}{f}$ l : lower class limit of 1st quartile class interval c.f. : preceding cumulative frequency of less than type of 1st quartile class interval h : width f : frequency of 1st quartile class interval
$Q_3 = l + \frac{\left(3*\frac{N}{4}-c.f.\right)*h}{f}$ l : lower class limit of 3rd quartile class interval c.f. : preceding cumulative frequency of less than type of 3rd quartile class interval h : width f : frequency of 3rd quartile class interval
Coefficient of Q.D. (C.Q.D.) = $\frac{Q_3-Q_1}{Q_3+Q_1}$

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- **Mean Deviation**

It is also known as average deviation. In this case deviation taken from any average especially Mean, Median or Mode.

While taking deviation we have to ignore negative items and consider all of them as positive.

Mean deviation from arithmetic mean (M.D. from A.M.) M.D.= $\frac{\sum_i f_i \lvert x_i - \bar{x} \rvert}{\sum_i f_i}$ $\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$
Coefficient of Mean deviation C.M.D. = $\frac{M.D.}{\bar{x}}$
Mean deviation from Median : M.D.= $\frac{\sum_i f_i \lvert x_i - Median \rvert}{\sum_i f_i}$ Median = $l + \frac{\left(\frac{N}{2} - c.f.\right) * h}{f}$ Coefficient of Mean deviation C.M.D. from Median = $\frac{M.D. from\ Median}{Median}$
Mean deviation from Mode : M.D.= $\frac{\sum_i f_i \lvert x_i - Mode \rvert}{\sum_i f_i}$ Mode = $l + \frac{(f_m - f_1.) * h}{2f_m - f_1 - f_2.}$ Coefficient of Mean deviation C.M.D. from Median = $\frac{M.D. from\ Mode}{Mode}$

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- **Standard Deviation:**

The concept of standard deviation was first introduced by **Karl Pearson in 1893.**

The standard deviation is the most useful and the most popular measure of dispersion.

Just as the arithmetic mean is the most of all the averages, the standard deviation is the best of all measures of dispersion.

The standard deviation is represented by the Greek letter $\boldsymbol{\sigma}$ **(sigma).**

It is always calculated from the arithmetic mean.

S.D. = $\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2}$ $\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$
Variance = σ^2 = S.D.^{2} $= \frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2$ $\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$
Coefficient of Variation   C.V.= $\frac{\text{S.D.}}{\bar{x}} * 100$

- Quartile deviation considers only 50% of the item and ignores the other 50% of items in the series.
- Mean deviation no doubt an improved measure but ignores negative signs without any basis.

- **Combined Standard deviation**

Let group of size n_1 having mean $\bar{x}_1$ and S.D. σ_1 is merged with another group of n_2, mean $\bar{x}_2$ and S.D. σ_2, then S.D. of the combined group of size $n_1 + n_2$ is given by

$$\sigma = \sqrt{\frac{n_1\left(\sigma_1^2+d_1^2\right)+n_2\left(\sigma_2^2+d_2^2\right)}{n_1+n_2}}$$

Where,

$$d_1 = \bar{x}_1 - \bar{x}\,; \qquad d_2 = \bar{x}_2 - \bar{x}\,; \qquad \bar{x} = \frac{n_1.\bar{x}_1+n_2.\bar{x}_2}{n_1+n_2}$$

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