Pigeon hole principle:

1. Let A= {1, 2, 3, 4, 5, 6, 7, 8}
   a. If five integers are selected from A, must at least one pair of the integer have a sum of 9?
   b. If four integers are sleeted from A, must at least one pair of the integer have a sum of 9?

2. Show that in a group of 85 people at least 4 must have same last initial.
3. If 5 cards are selected from standard 52-card deck, must at least 2 be of the same suite?
4. If ten integers are chosen from 1 to 20 inclusive, must at least one of them will be even.
5. How many integers you must pick to be sure at least two of them have the same reminder when divided by 15.
6. In a group of 2000 people must at least 5 have been born in same month. Why?
7. Given any set of 8 integers, must there be two that have the same reminder when divided by 8? Why?

Random Variable, Expectation and variance:

1) A person pays $1 to play the following game: The person tosses a fair coin four times. If no heads occur, the person pays an additional $2, if one head occurs, the person pays an additional $1, if two heads occur, the person just loses the initial dollar, if three heads occur, the person wins $3, and if four heads occur, the person wins $4. What is the person's expected gain or loss?

2) When a pair of balanced dice are rolled and the sum of the numbers showing face up is computed, the result can be any number from 2 to 12, inclusive. What is the expected value of the sum?

3) A lottery game offers $2 million to the grand prize winner, $20 to each of 10,000 second prize winners, and $4 to each of 50,000 third prize winners. The cost of the lottery is $2 per ticket. Suppose that 1.5 million tickets are sold. What is the expected gain or loss of a ticket?

Bayes' Theorem:

1. An urn contains 5 blue and 7 gray balls. Let us say that 2 are chosen at random, one after the other, without replacement.
   - **a.** Find the following probabilities and illustrate them with a tree diagram: the probability that both balls are blue, the probability that the first ball is blue and the second is not blue, the probability that the first ball is not blue and the second ball is blue, and the probability that neither ball is blue.
   - **b.** What is the probability that the second ball is blue?
   - **c.** What is the probability that at least one of the balls is blue?
   - **d.** If the experiment of choosing two balls from the urn were repeated many times over, what would be the expected value of the number of blue balls?

2. Two different factories both produce a certain automobile part. The probability that a component from the first factory is defective is 2%, and the probability that a component from the second factory is defective is 5%. In a supply of 180 of the parts, 100 were obtained from the first factory and 80 from the second factory.
   - **a.** What is the probability that a part chosen at random from the 180 is from the first factory?
   - **b.** What is the probability that a part chosen at random from the 180 is from the second factory?
   - **c.** What is the probability that a part chosen at random from the 180 is defective?
   - **d.** If the chosen part is defective, what is the probability that it came from the first factory?

3. A student taking a multiple-choice exam does not know the answers to two questions. All have five choices for the answer. For one of the two questions, the student can eliminate two answer choices as incorrect but has no idea about the other answer choices. For the other question, the student has no clue about the correct answer at all. Assume that whether the student chooses the correct answer on one of the questions does not affect whether the student chooses the correct answer on the other question.
   - **a.** What is the probability that the student will answer both questions correctly?
   - **b.** What is the probability that the student will answer exactly one of the questions correctly?
   - **c.** What is the probability that the student will answer neither question correctly?