Unit - II

BOOLEAN ALGEBRA & LOGIC GATES

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BOOLEAN THEORMS

Identity	Dual
Operations with 0 and 1: 1. X + 0 = X (identity) 3. X + 1 = 1 (null element)	2. X.1 = X 4. X.0 = 0
Idempotency theorem: 5. X + X = X	6. X.X = X
Complementarity: 7. X + X' = 1	8. X.X' = 0
Involution theorem: 9. (X')' = X	

BOOLEAN LAWS

Identities for multiple variables	
Cummutative law: 10. $X + Y = Y + X$	11. $X.Y = Y X$
Associative law: 12. $(X + Y) + Z = X + (Y + Z)$ $= X + Y + Z$	13. $(XY)Z = X(YZ)$ $= XYZ$
Distributive law: 14. $X(Y + Z) = XY + XZ$	15. $X + (YZ) = (X + Y)(X + Z)$
DeMorgan's theorem: 16. $(X + Y + Z + \ldots)' = X'Y'Z'\ldots$ or $\{f(X_1,X_2,\ldots,X_n,0,1,+,.)\}$ $= \{f(X_1',X_2',\ldots,X_n',1,0,.,+)\}$	17. $(XYZ\ldots)' = X' + Y' + Z' + \ldots$

De MORGAN’S THEORM

- **DeMorgan's** Theorems describe the equivalence between gates with inverted inputs and gates with inverted outputs.
- Simply put, a NAND gate is equivalent to a Negative-OR gate, and a NOR gate is equivalent to a Negative-AND gate.

De MORGAN’S THEORM

(i)Statement

The theorem states that the complement of sum of variables is equal to the product of their individual complements.

$\overline{A+B} = \overline{A}\,.\,\overline{B}$

Proof

A	B	$\overline{A}$	$\overline{B}$	$\overline{A+B}$	$\overline{A}\,.\,\overline{B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

(ii)Statement-

The theorem states that the complement of product of variables is equal to the sum of their individual complements.

$\overline{A.B} = \overline{A} + \overline{B}$

Proof

A	B	$\overline{A}$	$\overline{B}$	$\overline{A.B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

Another simple way of remembering the theorem is '**Cut the line and change the sign**'. De Morgan's law is used to simplify the Boolean expressions in digital circuits. De Morgan's laws can be applied to any number of variables.

E.g. De Morgan's laws for three variables

$\overline{A + B + C} = \overline{A} \,.\, \overline{B} \,.\, \overline{C}$ &

$\overline{A.B.C} = \overline{A} + \overline{B} + \overline{C}$

Perfect induction

- **perfect induction**, or the **brute force method**, is a method of mathematical proof in which the statement to be proved is split into a finite number of cases or sets of equivalent cases and each type of case is checked to see if the proposition in question holds.
- This is a method of direct proof. A proof by exhaustion contains two stages:
- A proof that the cases are exhaustive; i.e., that each instance of the statement to be proved matches the conditions of (at least) one of the cases.
- A proof of each of the cases.

Induction proof of $x+x'\cdot y=x+y$

Use perfect induction to prove $x+x'\cdot y=x+y$

x	y	$x'y$	$x+x'y$	$x+y$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equivalent

Reduction of logical expression using Boolean Algebra

- Boolean algebra finds its most practical use in the simplification of logic circuits.
- If we translate a logic circuit's function into symbolic (Boolean) form, and apply certain algebraic rules to the resulting equation to reduce the number of terms and/or arithmetic operations, the simplified equation may be translated back into circuit form for a logic circuit performing the same function with fewer components.
- If equivalent function may be achieved with fewer components, the result will be increased reliability and decreased cost of manufacture.

AB=1

$A + AB$

↓ Factoring **A** out of both terms

$A(1 + B)$

↓ Applying identity $A + 1 = 1$

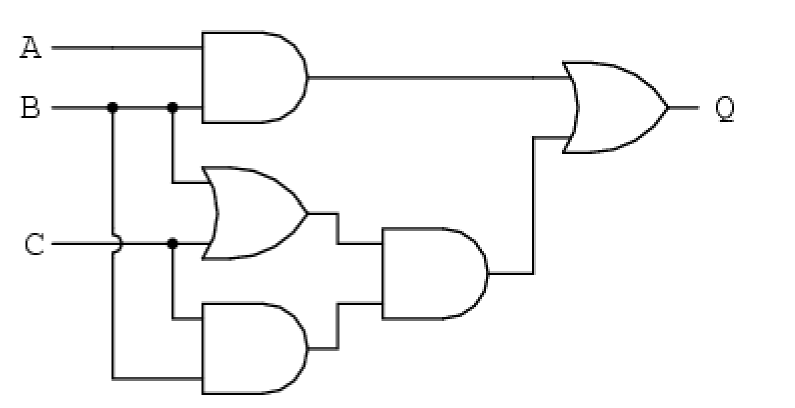
$A(1)$

↓ Applying identity $1A = A$

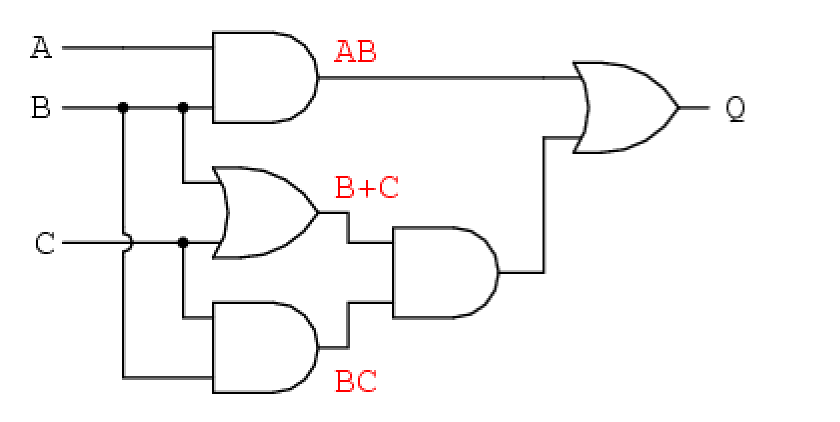
A

Derive Boolean Expression from circuit

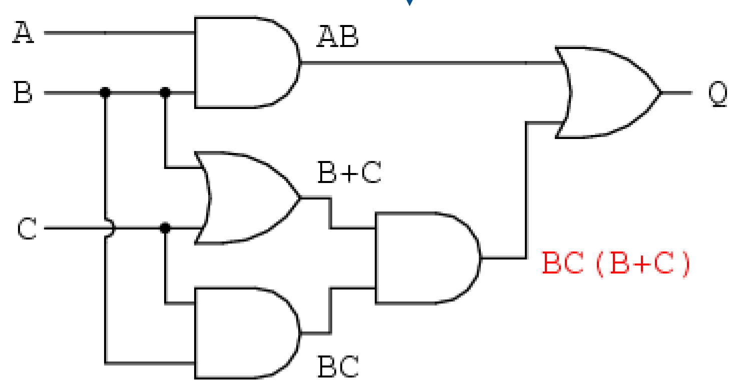
- To understand how to Derive Boolean Expression from circuit, let us consider following circuit.



Step -1



Step - 2



Step -3