Moments, Skewness, and Kurtosis

- Moments for Grouped Data
- Relations Between Moments
- Computation of Moments for Grouped Data,
- Charlie's Check and Sheppard's Corrections
- Moments in Dimensionless Form
- Skewness, Kurtosis

Moments: For any frequency distribution, the **r^th^** moment about any point **A** is defined as the Arithmetic mean of r^th^ powers of deviations from the point A.

Moments about mean (or Central Moments) :

Let $x_1, x_2, \ldots, x_n$ be the n values of the variable x, then the **r^{th}** moment about the mean (arithmetic mean) $\bar{x}$ is denoted by μ_r and is defined by

$$\mu_r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^r}{n}, for\ r = 0, 1, 2, 3 \ldots \ldots$$

For a frequency distribution. Let

x :	x_1	x_2	…	x_n
f:	f_1	f_2	…	f_n

be a discrete frequency distribution. Then the rth moment μr about the mean x is defined by

$$\mu_r = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})}{f_i}$$ **for r-0, 1, 2, 3, ….**

For all distribution $\mu_1 = 0$

For r=2, $$\mu_2 = \frac{1}{N}\sum f_i(x_i - \bar{x})2 = \sigma^2 = \text{variance.}$$

Hence for all distribution $\mu_2 = (\text{standard deviation})^2 = \text{Variance}$

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MOMENTS ABOUT ANY POINT (RAW MOMENTS)

For any frequency distribution the rth moment about any point x = A, is defined as the arithmetic mean of the **r^th^** powers of the deviations from the point x=A and is denoted by μ'_r

If

X :	**x1**	**x2**	**…**	**xn**
F:	**f1**	**f2**	**…**	**fn**

Be discrete frequency distribution, then

$$\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^r,\ r = 0,1,2,3,\ldots,\ \text{and}\ \sum_{i=1}^{n} f_i = N.$$

KARL PEARSON'S β AND γ COEFFICIENTS

Karl Pearson gave the following four coefficients. Calculated from the central moments, which are defined as

Beta coeficients	Gamma coefficients
$\beta_1 = \mu_3^2/\mu_2^3$	$\gamma_1 = \pm\sqrt{\beta_1}$
$\beta_2 = \mu_4/\mu_2^2$	$\gamma_2 = \beta_2 - 3 = \dfrac{\mu_4 - 3\mu_2^2}{\mu_2^2}$

The sign of γ_1 depends upon μ_3 is positive then γ_1 is positive. If μ_3 is negative then γ_1 is negative. The above four coefficients are pure numbers and thus do not have any unit. The β and γ coefficients give some idea about the shape of the curve obtained from the frequency distribution. This we shall discuss in the topic Kurtosis and Skew ness.

To Calculate Central Moments.

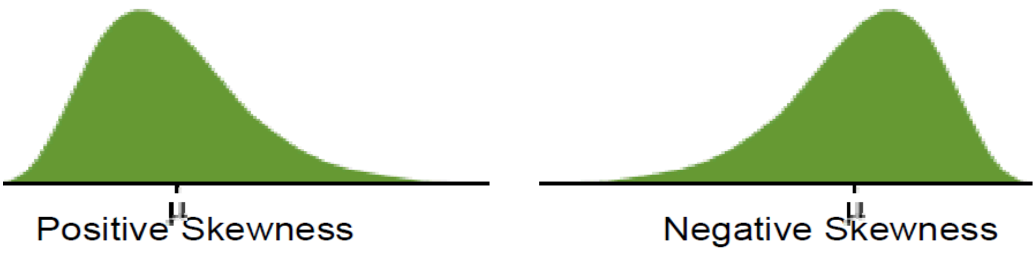
$\mu_1 = 0$ (always)

$\mu_2 = \mu_2' - (\mu_1')^2$

$\mu_3 = \mu_3' - 3\,\mu_2'\,\mu_1' + 2\,(\mu_1')^3$

$\mu_4 = \mu_4' - 4\,\mu_3'\,\mu_1' + 6\,\mu_2'(\mu_1')^2 - 3\,(\mu_1')^4$

SKEWNESS



Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point.

- Skew is a measure of symmetry in the distribution of scores.
- If $s^3 < 0$, then the distribution has a negative skew.
- If $s^3 > 0$ then the distribution has a positive skew.
- If $s^3 = 0$ then the distribution is symmetrical.
- The more different s^3 is from 0, the greater the skew in the distribution

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KURTOSIS

Kurtosis is a parameter that describes the shape of a random variable's probability distribution. Kurtosis characterizes the relative peakedness or flatness of a distribution compared to the normal distribution.

- Positive kurtosis indicates a relatively peaked distribution.
- Negative kurtosis indicates a relatively flat distribution.
- Kurtosis measures whether the scores are spread out more or less than they would be in a normal (Gaussian) distribution.
- When the distribution is normally distributed, its kurtosis equals to 3 and it is said to be **mesokurtic**
- When the distribution is less spread out than normal, its kurtosis is greater than 3 and it is said to be **leptokurtic**
- When the distribution is more spread out than normal, its kurtosis is less than 3 and it is said to be **platykurtic**

NOTE: Collectively, the variance (s^2), skew (s^3), and kurtosis (s^4) describe the shape of the distribution.

Coefficient of skewness $= \dfrac{Q_3 + Q_1 - 2M_d}{Q_3 - Q_1} = \dfrac{(Q_3 - M_d) - (M_d - Q_1)}{(Q_3 - M_d) + (M_d - Q_1)}$

Coefficient of skewness$= \dfrac{Mean - Mode}{S\tan dardDeviation} = \dfrac{M - M0}{\sigma}$

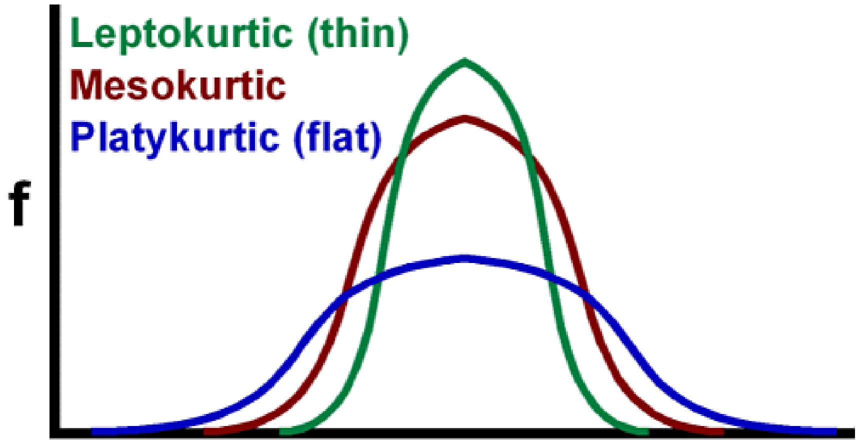
Coeff. Of skewness $= \dfrac{3(M - M_d)}{\sigma}$

Kurtosis or $\beta_2 = \mu_4 / \mu_2^{\,2}$.

$\gamma_2 = \beta 2 - 3 = \dfrac{\mu_4 - 3\mu_2^2}{\mu_2^2}$

Deductions.

(1) If $\gamma_2 = 0$, the curve is normal.
(2) If $\gamma_2 > 0$, the curve is leprokurtic.
(3) If $\gamma_2 < 0$, the curve is platykurtic.



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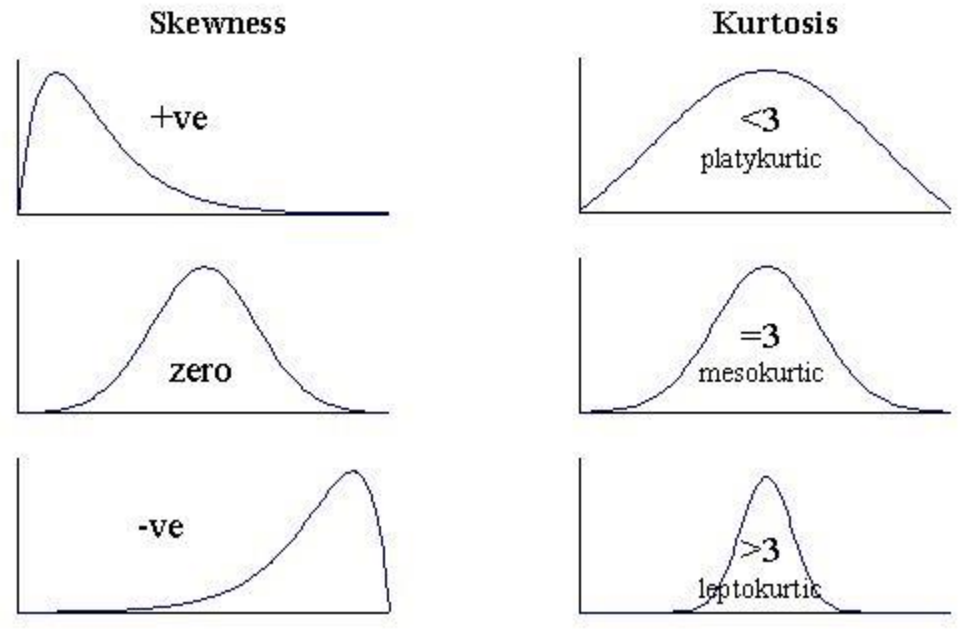

Sheppard's correction are approximate corrections to estimate of moments computed from binned data. The concept is named after William Fleetwood Sheppard.

Let $\mathbf{m_k}$ be the measured kth moments $\widehat{\mu_k}$ the corresponding corrected moment, and **c** the class interval, <u>no correction is necessary for the mean (first moments about zero).</u>

The first few measured and corrected moments about the mean are then related as follows:

$$\widehat{\mu_2} = m_2 - \frac{1}{12}c^2$$

$$\widehat{\mu_3} = m_3$$

$$\widehat{\mu_4} = m_2 - \frac{1}{2}\, m_2 c^2 + \frac{7}{240}\, c^4$$