<u>Random Variable:</u>

A random variable (r.v) X is a real valued function defined on the sample space S.

X: $\mathbb{N} \cup \{0\} \rightarrow S$ as X (n) = S_i, where $S_i \in S$ for n $\in \mathbb{N} \cup \{0\}$

There are two types of random variable

1. **Discrete random variable**
   It takes only integer value, like no. of students, no. of children in family etc.
2. **Continuous random variable**
   It takes real values, like height of a person, weight, income etc.

<u>Example</u>: Let S be the sample space for random experiment of tossing of two coins simultaneously and random variable X denotes no of head appeared.

S= {HH, HT, TH, TT}

Now X : No. of Head appeared

Then X : 0 if no head appeared

1 if 1 head appeared

2 if 2 head appeared

Hence to each sample points in S we have assigned a real number, which uniquely determine the sample point. The variable X is called as the **random variable** defined on the sample space S.

We can also find the probabilities of r.v.X as

p (no head appeared)= p(X=0)= 1/4

p (1 head appeared)=p(X=1)=2/4

p (2 head appeared)=p(X=2)=1/4

Hence **probability distribution of r.v.** X is given by

X=x	0	1	2
P(X=x)	¼	2/4	¼

A **Probability Distribution** is a listing of all possible numerical outcomes from a random variable.

- **Note:**

In general probability distribution of X satisfies the following conditions;

(i) all p(x) are positive. i.e. $p(x) \geq 0$ (ii) $\sum p(x) = 1$ for all x.

A Random Variable defined on sample space may be finite or infinite. r.v. are discrete (take integer value) or continuous (take real values)

- **Probability Mass Function:**

If X is discrete random variable with distinct values $x_1, x_2, x_3, \ldots, x_n$ then its corresponding probabilities $p(X= x_1)$, $p(X= x_2)$, $p(X= x_3)$,…. $P(X= x_n)$ are called probabilities mass function (p.m.s.)

If X is discrete random variable with distinct values $x_1, x_2, x_3, \ldots, x_n$ then the function p(x) defined as $p(X) = p(X=x_i)=p_i$ if $x= x_i$

$= 0$ otherwise

Is called probabilities mass function of r.v.X.

- **Probability density function(p.d.f.):**

  If X is continuous random variable with distinct values $x_1, x_2, x_3, \ldots, x_n$ (for $x \in (a, b)$) then the function f(x) defined as $f(X) = f(X=x_i)=f_i$ if $x= x_i$, $a < x < b$

  $= 0$ otherwise

  Is called probabilities density function of r.v.X.

- **<u>Expectation and Variance of random variable</u>**

Expectation:

Suppose a random variable X defined on sample space S takes values x1, x2,x3,----- xn. with respective probabilities p1, p2, p3, ----- pn ; P (x= x1) = p1, it‘s expected value is defined as,

a) $E(X) = \sum x.p(x)$ if x is discrete random variable
b) $E(X) = \int x.p(x)\,dx$ if x is continuous random variable

Note: Expected value is also called as the **mean** of X.

Variance of X :

V(X): For the random variable X, variance is given by,

$V(X) = E(X^2) - (E(X))^2$ $Where\ E(X^2) = \sum x^2 p(x)$, if x is discrete random variable

$V(X) = E(X^2) - (E(X))^2$ $Where\ E(X^2) = \int x^2 p(x)\,dx$, if x is continuous random variable

- **Properties of E(X) and V(X);**
  1. E(ax+b)= aE(x)+ b
  2. V(ax+b)= a^2V(X)
  3. V(b)=0

DISTRIBUTION

- **Uniform Distribution:**

Let X be a discrete random variable follows uniform distribution then its probability density function is given by

P (X=x) = 1/n ; x= 0, 1, 2 ….n

0 O.W.

OR

Let X be a **continuous random** variable follows Uniform distribution then its probability density function is given by

P(X=x) = 1 / (b-a) ; a<x<b

0 ;O.W.

Mean of Uniform Distribution = E (X)= $\frac{a+b}{2}$

Variance of Uniform Distribution =V (X)= $\frac{(b-a)^2}{12}$

Note:

In this distribution every possible outcome has an equal chance, or likelihood, of occurring (1 out of the total number of outcomes).

- **Binomial Distribution:**

Let X be a random variable follows binomial distribution with parameter n, p (n: sample size and p; probability of success of X) then its probability density function is given by

P(X=x) = $^nC_x\ p^x\ q^{(n-x)}$; x= 0, 1, 2, …, n and q=1-p

= 0 ; O.W.

Mean of Binomial distribution= np

Variance of Binomial distribution = npq

- **Poisson's Distribution:**

Let X be a random variable follows poisson's distribution with parameter λ then its probability density function is given by

P (X=x) = $\frac{e^{-\lambda}\lambda^{x}}{x!}$; x= 0, 1, 2 …

0 ; O.W.

Mean of Poisson's Distribution = E (X) = λ

Variance of Poisson's Distribution = V(X) = λ

- **Exponential Distribution:**

Let X be a random variable follows exponential distribution then its probability density function is given by

P (X=x) =$e^{-\lambda}\lambda^{x}$; x= 0, 1, 2,

0 ; O.W.

- **Normal Distribution:**

Let X be a continuous random variable follows normal distribution with mean μ and variance σ^2 then probability density function is given by

$f(x) = \frac{1}{2\sqrt{\pi\sigma}}.e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$; $-\infty < x < \infty$; $-\infty < \mu < \infty$;

0 ;o.w.

Properties of the normal distribution:

1. It is bell shaped (and thus symmetrical) in appearance
2. It's measures of central tendency are all identical
3. It's "middle spread" is equal to 1.33 standard deviations. This means that the interquartile range is contained within an interval of two-thirds of a standard deviations below and above the mean.
4. It's associated random variable has an infinite range.

➢ Standard Normal distribution

If z = $\frac{x-\mu}{\sigma}$ then z is said to follow normal distribution with mean 0 and variance 1 then probability density function is given by

$$f(z) = \frac{1}{2} e^{\frac{-z^2}{2}}$$

➢ Bernouli Distribution:

Consider a series of n independent Bernouli trials, where p: probability of success and q: probability if failure.

Let random variable X denotes number of success in "n" trials, to find probability of X=x is nothing but finding probability of getting x success and (n-x) failures

$P(X=x) = \mathbf{p^x\ q^{(n-x)}}$; x= 0, 1, 2, …, n and q=1-p

= 0 ; O.W.

➢ t-distribution:

If a random variable X is normally distributed, then the following statistic has a t distribution with n-1 **degrees of freedom**: $\boldsymbol{Z = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}}$

- The t-distribution looks much like the normal distribution except that it has more areas in the tails and less in the center.
- Because sigma is unknown and we are using S to estimate it, then we are more cautious in our inferences, the t-distribution takes this into account.
- As the sample size increases, so do the degrees of freedom, and so the t-distribution becomes closer and closer to the normal distribution