UNIT-V: DECISION THEORY

Decision theory provides a scientific/logical method for decision making. It may be defined as a process which results in the selection of the best possible option from a set of alternatives available to the Decision maker. The best selected action is considered to meet the objectives of the decision problem more satisfactorily than others according to the decision maker. Some of the decisions making situations may be as listed below.

1) A students has to select a course/stream for study keeping focus on job opportunities in mind.
2) A bakery owner has to decide on the number of loves to be prepared on a day.
3) A departmental store has to decide on the stock of T.V. sets for the month against the uncertain demand.
4) A company has to decide on the production size of low, moderate or high against the possible demand in the market so as to make maximum profit with minimum opportunity loss of profit.

Terminology:

1. **Course of Action-** Course of action refers to the alternatives available with the decision maker. For example a student may have an option to select subjects of his choice like English/Hindi/Marathi. Decision maker has complete control on the action to be selected.

2. **States of Nature-** It refers to the possible response to the action selected/executed by the Decision maker. S(h)e has no control over the response or state of nature.

3. **Pay-**off- Payoff is the effect of the action when responded by the state of nature observed.

Construction of Sample decision problem:

A departmental store buys Christmas cakes in the lots of 50 only. Each piece of cake costs Rs.10 is sold at Rs 25. Unsold cakes are total loss to the store. The probability distribution of the possible demand for the cakes based on the past experience is as follows,

Demand:	**50**	**100**	**150**	**200**
P(Demand):	**20%**	**40%**	**30%**	**10%**

Prepare a payoff table for the profit earned

Solution: From the given information, we list out the strategies(course of action) available to the store management. It is assumed that, the management will not keep the stock below the minimum demand

or more than the maximum demand observed in past, there will be four course of actions (Stocks). They are:

A1: by 50 cakes A2: by 100 cakes

A3: by 150 cakes A4: by 200 cakes

Similarly, we can list out the states of nature(Events) in the problems as the possible demand. They are:

S1: Demand will be of 50 cakes

S2: Demand will be of 100 cakes

S1: Demand will be of 150 cakes

S1: Demand will be of 200 cakes

Now, we calculate the payoff values for every course of action against the possible demand.

We know, cost price of the cake =10Rs & Selling price = 25Rs. Therefore, Profit= 25-10 =15Rs. per piece

When a piece of cake is not sold there is a loss of RS. 10

i.e profit = -10Rs. Per unsold piece

Therefore we have, for the calculation of payoffs,

Profit = 15Rs.per piece sold if demand(D) ≥stock(S)

& = -10per piece if demand< stock.

With this information we can calculate the payoff for the

different course of actions as,

payoff =15 x Demand if demand ≥ stock

=15x Demand -10x(Stock-Demand) if demand < stock

For action A1; Stock= 50

States of nature S1; payoff =15x50 =750

States of nature S2; payoff = 15x100=1500 & so on.

For action A2; Stock =100

For S1 i.e. Demand =50 (< Stock)

Payoff = 15x50-10x(100-50)=750-500=250

For S2 i.e. Demand =100

Payoff = 15x100=1500 & so on

Hence the payoff table will be as follows,

States of Nature	***Courses of Action***			
	$A_{1(50)}$	$A_{2(100)}$	$A_{3(150)}$	$A_{4(200)}$

S1(50)	*750*	*250*	*-250*	*-500*
S2(100)	*750*	*1000*	*500*	*0*
S3(150)	*750*	*1000*	*2250*	*1750*
S4(200)	*750*	*1000*	*2250*	*3000*

This is the decision problem constructed for the store keeper.

Solution to Decision Problem:

Optimal decision is selected using the two basic criteria,

A. Decision under uncertainty-

Under this situation the decision maker has no information on the state of nature. For example manufacturer has no predictions on market demand. The optimal decision is obtained using the 4 criteria given below.

1. Maximax- In this method the Maximum of the maximum pay-off for the course of action is selected as the optimal decision. Hence first select Max pay-off for the action and then select the maximum from these maximum pay-offs. The course of this max(max pay-off) is the best action or optimal decision.

2. Maximin- In this method the Maximum of the mimimum pay-off for the course of action is selected as the optimal decision. Hence first select Max pay-off for the action and then select the maximum from these maximum pay-offs. The course of this max(max pay-off) is the best action or optimal decision.

3. Laplace (Average) criteria- Under this criteria the total payoff or average payoff for each action is calculated and action with maximum average is taken as the best action or optimal decision.

4. Minimax Regret criteria-In this criteria the maximum regret/opportunity lass is minimized for the action, opportunity loss or regret value can be explained as below.

Regret Value- It is defined as the amount of payoff lost by not taking the action with highest payoff under the observed state of nature.

Students should note that, regret is calculated after the state of nature has occurred.

After the regret values are obtained the optimal decision is that which produces minimum opportunity loss/regret value.

A. Decision under risk (probability values are given)

In this situation the decision maker has some information/prediction on the state of nature. For example manufacturer has little information on market demand like Good/Moderate/Poor in terms of chance values %.. The optimal decision is obtained using the 2 criteria given below.

1. EMV (Expected Monetary Value) Criteria

EMV is calculated by the formula,

$$EMV=\sum xp(x)$$ where, p(x) = probability of payoff x

2. EOL (Expected Opportunity Loss) Criteria

$$EMV=\sum xp(x)$$ where, p(x) = probability of payoff x

SOLVED EXAMPLES

2.a.i.1. Solve the Decision problem using i) Maximax ii) Maximin & iii) Laplace criteria

Action/ States of nature	**A1**	**A2**	**A3**	**Optimal Decision**
S1	50	10	100	
S2	700	500	60	
S3	500	900	80	
Max	700	MAX 900	100	Maximax-A2
Min	50	10	MAX 60	Maximin-A3
Average	416.667	MAX 470	80	Max(Ave)-A2

i) Maximum of Maximum payoff = 900 for action A2

ii) Maximum of Minimum payoff = 60 for action A3

iii) Maximum of (Average) payoff = 470 for action A2

2.a.i.2. Solve the Decision problem using i) Maximax ii) Maximin & iii) Laplace criteria

Pay-off Table					Maximax	Maximin
Stock/Demand	**90**	**100**	**110**	**120**	**Max**	**Min**
A1- 90	2250	2250	2250	2250	2250	MAX 2250
A2- 100	1000	2500	2500	2500	2500	1000
A3- 110	-250	1250	2750	2750	2750	-250
A4- 120	-1500	0	1500	3000	MAX 3000	-1500
				Stock	A4	A1

From the above table we get the optimal decision under,

i) Maximum of Maximum payoff = 900 for action A2- Optimal decision

ii) Maximum of Minimum payoff = 60 for action A3-Optimal decision

iii) Maximum of (Average) payoff = 470 for action A2- Optimal decision

1. Solve the Decision problem using EMV criteria

Pay-off Table	
Action	**State of nature** **S1** **S2** **S3**
A1	(x)= 20 5 -1
A2	8 5 4
A3	-10 5

	10
Probability	p(x) =0.3 0.4 0.3

We calculate the EMV values as follows

EMV(A1) = 20x0.3 +5x0.4 +(-1)x0.3=7.7--Max

EMV (A2) = 8x0.3 +5x0.4 + 4x0.3 = 5.6

EMV (A3) = -10x0.3 +5x0.4 +10x0.3= 2

Hence, the optimum decision is A1

2. Solve the decision problem using, Minimax Regret criteria

Pay-off Table

Action/ States of nature	**A1**	**A2**
S1	50	10
S2	**700**	500
S3	500	**900**

Solution: For the given payoff tab le we first create the Regret table as sh

Pay-off Table				**Regret Table** **Mark max for the States of nature**	
Action/ States of nature	**A1**	**A2**	**A3**	**A1**	**A2**

S1	50	10	**100 Max**	100-50=50	90
S2	**700 Max**	500	60	700-700=0	200 Max
S3	500	**900 Max**	80	900-500=400 Max	0
Max regret				400 Max	**200 Min(Max)**

3. Solve the Decision problem using EOL criteria

Pay-off

Action	**State of nature (Mark max)**		
	S1	**S2**	**S3**
A1	20Max	0	11Max
A2	8	0 Max	6
A3	-10	0	0

Regret (OL) table
(Mark max for the state of nature)

Action	**State of nature**		
	S1	**S2**	**S3**
A1	20-20 = 0	0	11-11=0
A2	20-8= 12	0	11-6=5
A3	20-(-10)=30	0	11-0-11

EOL (A1) = 0x0.3+0x0.4+11x0.3 =3.3----Min EOL Optimal Decision: A1

EOL (A2) = 12x0.3+0x0.4+6x0.3 =5.4

EOL (A3) = 30x0.3+0x0.4+11x0.3 = 12.3

EXAMPLES FOR PRACTICE

1. Solve decision problem using Maxmin & Laplace criterion:
(6)

Action → Event ↓	**A1**	**A2**	**A3**
E1	10	20	36
E2	16	44	16
E3	42	36	24

2. Determine the best decision according to EMV criterion.

Action/Events	S1	S2	S3
A	10	12	25
B	18	24	10
C	25	30	20
Probability:	0.2	0.5	0.3

3. Draw a Decision Tree diagram & solve the decision problem using, EMV criteria.

Action/States of nature	S1	S2	S3
A1	50	70	75
A2	40	50	90

4. Obtain the optimal decision by Minimax criteria for the payoff tables below:

COURSE OF ACTION → States of NATURE ↓	**A**	**B**	**C**
S1	15	25	20
S2	25	12	18
S3	10	16	15
S4	20	15	10

a.

Event → Action ↓	**E1**	**E2**	**E3**
A1	5	10	18
A2	8	22	8
A3	21	18	12
A4	30	7	19

b.

1. Given the pay-off matrix, solve the decision problem using,

i) Laplace ii) Maximin iii) Maximax

Action/States of nature	S1	S2	S3
A1	50	700	500
A2	10	500	900
A3	100	60	80

b) Determine the best decision according to EMV criterion.

Action	Events		
	E1	E2	E3
A1	10	12	25
A2	18	24	10
A3	25	30	20
Probability:	0.2	0.5	0.3

2. Draw a decision tree for the decision problem below and state the best possible decision.

Action/States of nature	E1	E2	E3
A1	100	350	-100
A2	150	50	150

3. a) Given the pay-off matrix, solve the decision problem using, Maximin & Maximax criteria

Action/Event	**E1**	**E2**	**E3**
A1	10	25	16
A2	25	12	18
A3	30	22	15
A4	15	25	20

b) Solve decision problem using EOL criterion:

Event→ Action↓	**E1**	**E2**	**E3**
A	5	10	18
B	8	22	8
C	21	18	12
P(E)	35%	25%	45%

4. Draw a decision tree for the decision problem below and state the best possible decision.

Product/Market demand	Poor	Average	Good
P	100	350	100
Q	150	50	150
P(Demand)	30%	55%	15%

Example based on EMV criteria:

Ex: Obtain the optimum decision for the given payoff matrix using expected monetary value (EMV) criteria.

States of Nature	***Courses of Action***		
	A_1	A_2	A_3
S1	*30*	*42*	*35*
S2	*60*	*18*	*30*
S3	*20*	*60*	*40*
S4(200)	*20*	*-60*	*-80*

Solution:

We calculate the expected monetary values (EMV) by the formula,

EMV = ∑payoff x P(States of nature)

For action A1: EMV= 30x0.2 +60x0.35+20x0.35+20x0.1 = 36*max

For action A2: EMV=42x0.2 +18x0.35+60x0.35-60x0.1 =29.7

For action A3: EMV= 35x0.2 +30x0.35+40x0.35-80x0.1 =23.5

The max(EMV)= 36 for the course of action A1

Hence on EMV criteria optimum decision is action A1.

Ex: A departmental store buys Christmas cakes in the lots of 50 only. Each piece of cake costs Rs. 10 is sold at Rs. 25. Unsold cakes are total loss to the store. The probability distribution of the possible demand for the cakes based on the past experience is as follows,

Demand:	**50**	**100**	**150**	**200**
P(Demand):	**0.2**	**0.4**	**0.3**	**0.1**

a) Prepare a payoff table for the profit earned

b) How much quantity the store should keep in stock in order to,

i) maximize the profit? Ii) minimize the opportunity loss?

Solution: From the given information, we list out the strategies(course of action) available to the store management.

Since the management will not keep the stock below the minimum demand or more than the maximum demand observed in past, there will be four course of actions (Stocks). They are:

A1: by 50 cakes A2: by 100 cakes

A3: by 150 cakes A4: by 200 cakes

Similarly, we can list out the states of nature(Events) in the problems as the possible demand. They are:

S1: Demand will be of 50 cakes

S2: Demand will be of 100 cakes

S1: Demand will be of 150 cakes

S1: Demand will be of 200 cakes

Now we calculate the payoff values for every course of action against the possible demand.

We know, cost price of the cake =10Rs & Selling price = 25Rs

Therefore, Profit= 25-10 =15Rs. Per piece

When a piece of cake is not sold there is a loss of RS. 10

i.e profit = -10Rs. Per unsold piece

Therefore we have, for the calculation of payoffs,

Profit = 15Rs.per piece if demand(D) ≥stock(S)

& = -10per piece if demand< stock.

With this information we can calculate the payoff for the

different course of actions as,

payoff =15 x D if demand ≥ stock

=15x D -10x(S-D) if demand < stock

For action A1; Stock= 50

States of nature S1; payoff =15x50 =750

States of nature S2; payoff = 15x100=1500 & so on.

For action A2; Stock =100

For S1 i.e. Demand =50 (< Stock)

Payoff = 15x50-10x(100-50)=750-500=250

For S2 i.e. Demand =100

Payoff = 15x100=1500 & so on

Hence the payoff table will be as follows,

States of Nature	***Courses of Action***			
	$A_{1(50)}$	$A_{2(100)}$	$A_{3(150)}$	$A_{4(200)}$
S1(50)	*750*	*250*	*-250*	*-500*
S2(100)	*750*	*1000*	*500*	*0*

S3(150)	*750*	*1000*	*2250*	*1750*
S4(200)	*750*	*1000*	*2250*	*3000*

Now we calculate the expected monetary values(EMV) as follows,

EMV = ∑payoff x P(States of nature)

For action A1: EMV= 750x0.2 +750x0.3+750x0.4+750x0.1 = 750.

For action A2: EMV= 250x0.2 +1000x0.3+1000x0.4+1000x0.1 = 850.

For action A3: EMV= -250x0.2 +500x0.3+2250x0.4+2250x0.1 = 1225*

For action A4: EMV= -500x0.2 +0x0.3+1750x0.4+3000x0.1 = 900.

Since the expected payoff of action A3 is maximum, the departmental store should keep the stock of 150 cakes.

Example based on EOL criteria:

EX: A decision problem has been expressed in the following payoff table:

Action			Event	
	I	II	III	IV
A	25	45	15	20
B	35	15	45	-30
C	30	25	35	-50
Probability	0.2	0.4	0.3	0.1

Solve the decision problem by EOL criteria.
Also calculate the expected value of perfect information(EVPI)
& comment on your result.

Solution: To solve the decision problem by EOL criteria we calculate the opportunity loss (regret) table as follows,

Regret(Event)= Best pay-off for the state of nature –Actual pay-off for the action.

e.g. For Event I, Regret = Best payoff 45 – Actual payoff of the action

Regret Table

Action	Event		
	I	II	III
A	**35-25 =10**	**0**	**30**
B	**0**	**30**	**0**
C	**5**	**20**	**10**
PROBABILITY	**0.2**	**0.4**	**0.3**

From the table we calculate the expected opportunity loss

EOL = ∑regret x P(States of nature)

For action A: EOL= 10x0.2 +0x0.4+30x0.3+0x0.1 =11*Min

For action B: EOL= 0x0.2 +30x0.4+0x0.3+50x0.1 =17.

For action C: EOL= 5x0.2 +20x0.4+10x0.3+70x0.1=19.