1) Assume that k is a particular integer,

a. Is -17 an odd integer? **b.** Is 0 an even integer? **c.** Is 2k-1 is odd?

2) Assume that m and n are particular integers.

a. Is 6m+8n even? **b.** Is a0mn+7 odd?

c. If m > n, is m^2+n^2 composite?

3) Prove that there is a perfect square that can be written as a sum of two other perfect squares.

4) Disprove the following statements by giving a counterexample.

a. For all real numbers a and b, if a < b then $a^2 < b^2$.

b. For all integers n, if n is odd then n−1 2 is odd.

c. For all integers m and n, if 2m+n is odd then m and n are both odd.

5) The sum of any two even integers are even.

6) The product of any two even integers are even.

7) The difference of any two even integers are even.

8) The difference between any odd integer and any even integer is odd.

9) The difference of any odd integer minus any even integer is odd.

10) The difference of any even integer minus any odd integer is odd.

11) For all integers n, if n is odd then 3n+5 is even.

12) If k is any odd integer and m is any even integer, then, k^2 +m2 is odd.

13) Prove that the following statements are false.

a. There exists an integer $m \geq 3$ such that $m^2 - 1$ is prime.

b. There exists an integer n such that $6n^2 + 27$ is prime.

c. There exists an integer $k \geq 4$ such that $2k^2 - 5k + 2$ is prime.

14) Determine whether the statement is true or false:

a. The product of any two odd integers is odd.

b. The negative of any odd integer is odd.

c. The difference of any two odd integers is odd.

d. The product of any even integer and any integer is even.

e. If a sum of two integers is even, then one of the summands is even.

f. For all integers n and m, if n−m is even then $n^3 - m^3$ is even.

g. For all integers n, if n is prime then $(-1)^n = -1$.

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15) True or false? If m is any even integer and n is any odd integer, then $m^2 + 3n$ is odd.

16) True or false? If a is any odd integer, then $a^2 + a$ is even.

17) True or false? If k is any even integer and m is any odd integer, then $(k+2)^2 - (m-1)^2$ is even.

18) For any rational numbers r and $s^2 r + 3s$ is rational.

19) If r is any rational number, then $3r^2 - 2r + 4$ is rational.

20) For any rational number s, $5s^3 + 8s^2 - 7$ is rational.

21) Answer the following:

a. Is 52 divisible by 13?

b. Does 7|56?

c. Does 5|0?

d. Does 3 divide (3k+1)(3k+2)(3k+3)?

e. 6m(2m+10) divisible by 4?

f. Is 29 a multiple of 3?

g. Is −3 a factor of 66?

h. Is 6a(a+b) a multiple of 3a?

22) For all integers a, b,and c,if a|bc then a|b or a|c.

23) For all integers a and b, if a|b then $a^2|b^2$.

24) For all integers a and n, if $a|n^2$ and $a \leq n$ then a|n.

25) For all integers a and b, if a|10b then a|10 or a|b

26) For all integers a, b, and c, if a|b and a|c then a|(b+c).

27) For all integers a, b,and c, if a|b and a|c then a|(b−c).

28) Use the unique factorization theorem to write the following integers in standard factored form. :

1,176 5,733 3,675

29) Evaluate the expressions

a. 43 div 9

b. 43 mod 9 8.

c. 50 div 7

d. 50 mod 7 9.

e. 28 div 5

f. 28 mod 5 10.

g. 30 div 2

h. 30 mod 2

30) Check the correctness of formula for the following values of DayT and N.

a. DayT= 6 (Saturday) and N =15

b. DayT=0 (Sunday) and N =7

c. DayT=4 (Thursday) and N =12

31) If today is Tuesday, what day of the week will it be 1,000 days from today?

32) January 1, 2000, was a Saturday, and 2000 was a leap year. What day of the week will January 1, 2050, be?

33) Show that any integer n can be written in one of the three forms n =3q or n =3q +1 or n =3q +2 for some integer q.

34) For all integers m, $m^2 =5k$, or $m^2 =5k+1$, or $m^2 =5k+4$ for some integer k.

35) The fourth power of any integer has the form 8m or 8m+1 for some integer m.

36) The product of any four consecutive integers is divisible by 8.

37) The square of any integer has the form 4k or 4k+1 for some integer k.

38) For any integer n, $n^2 +5$ is not divisible by 4.

39) The sum of any four consecutive integers has the form 4k+2 for some integer k.

40) For any integer n, $n(n^2 -1)(n+2)$ is divisible by 4.

41) Prove that for all integers n, if n mod5=3 then n^2 mod 5=4.

42) Prove that for all integers m and n, if m mod5=2 and n mod3=6 then mn mod 5=1.

43) Prove that for all integers a and b, if a mod7=5 and b mod7=6 then ab mod 7=2.

44) $1+3\sqrt{2}$ is irrational

45) For any integer a and any prime number p, if $p|a$ then $p\nmid (a+1)$.

46) prove that $\sqrt{2}$ is an irrational number

47) Prove that $\sqrt{3}$ is irrational

48) Prove that $\sqrt{2}+\sqrt{3}$ is irrational

49) Consider the following algorithm segments:

a. if x > 2

b. y :=0 then y := x +1 if x > 2 then y :=2x else do x := x −1 y :=3·x end do

What is the value of y after execution of these segments for the following values of x? i. x =5 ii. x =2

50) Trace the execution of the following algorithm segment by finding the values of all the algorithm variables each time they are changed during execution:

```
i :=1,s :=0
while (i ≤2)
s :=s+i
i :=i +1
 end
```

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51) Find the values of a and e after execution of the loops

a.
```
a :=2
for i :=1 to 2
a := a /2 + 1 /2
```

b.
```
e:= 0
f:= 2
for j=1:4
f:=f.j
e:= e+1/f
next j
```

52) Use the Euclidean algorithm to hand-calculate the greatest common divisors of each of the pairs of integers

a. 1,188 and 385 14. 509

b. 1,177 15. 832 and

c. 10,933 16. 4,131 and 2,43

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